HOT PROBLEMS

(Problems for developing Higher Order Thinking Skill)

Textual

1. If the sum of p terms of an A. P. is equal to the sum of q terms then show the sum of its p + q terms is zero.
2. How many two digit numbers leave the remainder 1 when divided by 5?

Solutions

1. **If the sum of p terms of an A. P. is equal to the sum of q terms then show the sum of its p + q terms is zero.**

Solution :

Given : Sum of p terms of an A.P = sum of q terms.

Prove : p + q terms = 0

Proof : In the given A.P.

Let the 1st term $t_1 = a$

Let common difference d = d

$S_p = S_q$ --- (Given)

$\therefore \quad S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore \quad \frac{p}{2}[2a + (p-1)d] = \frac{q}{2}[2a + (q-1)d]$

$p[2a + (p-1)d] = q[2a + (q-1)d]$ -- (cancelling $\frac{1}{2}$ on both sides)

$p[2a + (p-1)d] - q[2a + (q-1)d] = 0$

$2ap - 2aq + p(p-1)d - q(q-1)d = 0$

$2a(p-q) + (p^2 - p)d - (q^2 - q)d = 0$

$2a(p-q) + [p^2 - p - q^2 + q]\,d = 0$

$2a(p-q) + [p^2 - q^2 - p + q]\,d = 0$

$2a(p-q) + [(p-q)(p+q) - (p-q)]\,d = 0$

$2a(p-q) + [(p-q)(p+q-1)\,d] = 0$

$(p-q)\,2a + (p+q-1)\,d = 0$

$2a + (p+q-1)\,d = 0$ .....(1)

$S_{p+q} = \frac{p+q}{2}[2a + (p+q-1)d]$ .....(2)

$= \frac{p+q}{2}[0]$ .....(from 1 and 2)

$= 0$

$\therefore \quad S_{p+q} = 0$

2. How many two digit numbers leave the remainder 1 when divided by 5?

Solution :

Two digit nos. divided by 5 and leaving remainder 1.

1^{st} 2 digit no. divisible by 5 $= 10 \qquad (5 \times 2 = 10)$

The last 2 digit no. divided by 5 $= 95 \qquad (5 \times 19 = 95)$

The 2 digit nos. leaving remainder 1 when divided by 5:

The first 2 digit no . $= 10 + 1 = 11$

The last 2 digit no. $= 95 + 1 = 96.$

$\therefore \quad a = t_1 = 11 , \qquad t_n = 96, \qquad d = 5.$

$$t_n = a + (n - 1)\, d$$

$$96 = 11 + (n - 1)\, 5$$

$$96 - 11 = 5n - 5$$

$$85 = 5n - 5$$

$$5n = 85 + 5$$

$$5n = 90$$

$$n = \frac{90}{5}$$

$$\therefore \quad n = 18$$

∴ There are 18 two digit nos. divisible by 5 and leaving remainder 1.