

CHAPTER – 1

BASIC PROPORTIONALITY THEOREM

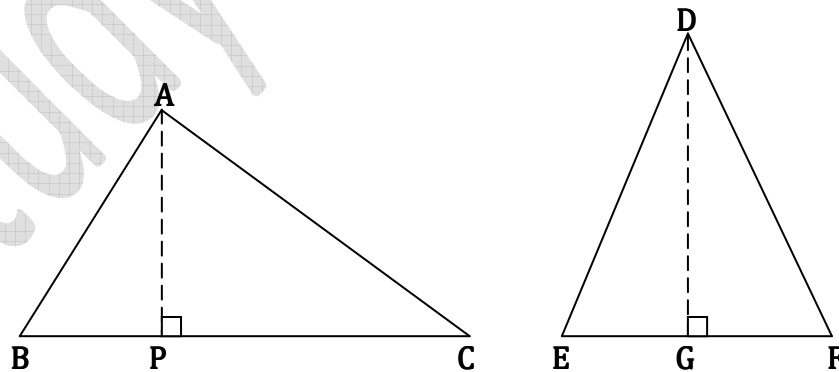
Ratio of areas of triangles :

The areas of triangles are proportional to the product of bases and corresponding heights of the triangles.

(Textual Solved Example)

Ex. 1 In fig. 1.9 in $\triangle ABC$, Seg $AP \perp$ seg BC and in $\triangle DEF$, sig $DG \perp$ seg EF .

$BC = 8$, $EF = 5$, $AP = 4$, $DG = 6$, Find $\frac{A(\triangle ABC)}{A(\triangle DEF)}$



Solution :

Given : In $\triangle ABC$, Seg $AP \perp$ seg BC

In $\triangle DEF$, seg $DG \perp$ seg EF .

$$BC = 8, EF = 5, AP = 4, DG = 6,$$

Find : $\frac{A(\triangle ABC)}{A(\triangle DEF)}$

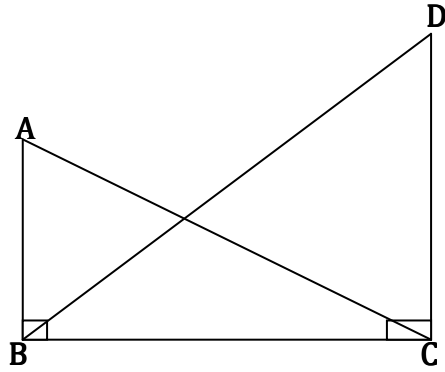
You know that, the ratio of area of two triangles is equal to the ratio of the product of their bases and corresponding height.

$$\begin{aligned}\therefore \frac{A(\triangle ABC)}{A(\triangle DEF)} &= \frac{BC \times AP}{EF \times DG} \\ &= \frac{8 \times 4}{5 \times 6} \\ &= \frac{32}{30} \\ &= \frac{16}{15}\end{aligned}$$

EXERCISE - 1.1

Textual

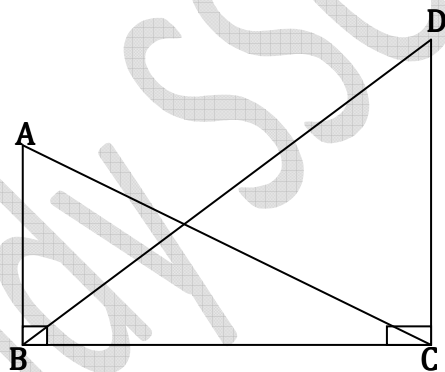
1. $\angle ABC = \angle DCB = 90^\circ$, $AB = 10$ and $DC = 15$. Find $\frac{A(\triangle ABC)}{A(\triangle DCB)}$



EXERCISE – 1.1

Solutions

1. $\angle ABC = \angle DCB = 90^\circ$, $AB = 10$ and $DC = 15$. Find $\frac{A(\triangle ABC)}{A(\triangle DCB)}$



Given : $\angle ABC = \angle DCB = 90^\circ$,

$AB = 10$ and $DC = 15$.

Find : $\frac{A(\triangle ABC)}{A(\triangle DCB)}$

Solution :

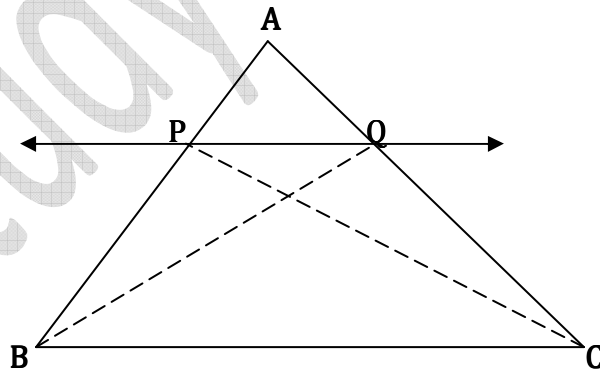
$$\frac{A(\triangle ABC)}{A(\triangle DCB)} = \frac{BC \times AB}{BC \times DC} = \frac{10}{15} = \frac{2}{3}$$

$$\therefore \frac{A(\triangle ABC)}{A(\triangle DCB)} = \frac{2}{3}$$

If bases are same then areas are in proportion to their corresponding heights.

Basic Proportionality Theorem (B. P. T) (Thale's Theorem) :

Theorem : If a line drawn parallel to one side of a triangle, intersects the other two sides in two distinct points, then that line divides the sides in proportion.



Given : In $\triangle ABC$, $A-P-B$, $A-Q-C$ and line $PQ \parallel$ side BC .

Show : $\frac{AP}{PB} = \frac{AQ}{QC}$

Construction : Draw seg BQ and CP.

Proof : Consider $\triangle APQ$ and $\triangle PQB$.

$$\frac{A(\triangle APQ)}{A(\triangle PQB)} = \frac{AP}{PB} \quad \text{--- (1) } (\triangle\text{s having same height})$$

Similarly consider $\triangle APQ$ and $\triangle PQC$

$$\frac{A(\triangle APQ)}{A(\triangle PQC)} = \frac{AQ}{QC} \quad \text{--- (2) } (\triangle\text{s having same height})$$

For $\triangle PQB$ and $\triangle PQC$ we have,

$$A(\triangle PQB) = A(\triangle PQC) \quad \text{-- (3) } \left(\begin{array}{l} \triangle\text{s having same base,} \\ \text{same height} \end{array} \right)$$

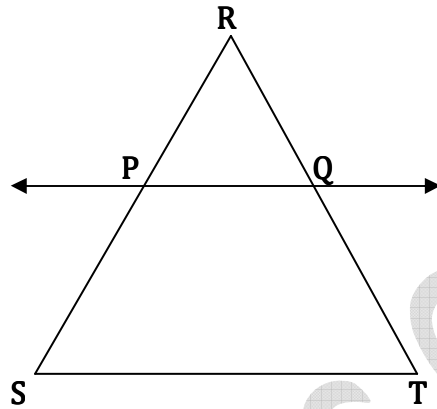
\therefore From (iii) we have

$$\frac{A(\triangle APQ)}{A(\triangle PQB)} = \frac{A(\triangle APQ)}{A(\triangle PQC)}$$

$$\therefore \frac{AP}{PB} = \frac{AQ}{QC}$$

(Textual Solved Example)

Ex.1 In the fig. line $PQ \parallel \text{seg } ST$, $R-P-S$ and $R-Q-T$, $RP = 4$, $PS = 6$, $RQ = 3$,
then find QT .



Solution :

In $\triangle RST$,

line $PQ \parallel \text{seg } ST$

$$\therefore \frac{RP}{PS} = \frac{RQ}{QT} \quad \text{--- (By B.P.T)}$$

$$\therefore \frac{4}{6} = \frac{3}{QT}$$

$$\therefore 4 \times QT = 6 \times 3$$

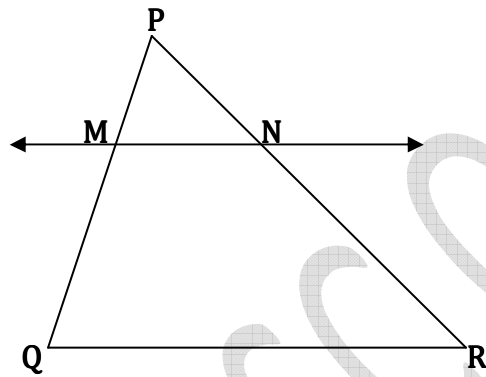
$$\therefore QT = \frac{6 \times 3}{4}$$

$$\therefore QT = 4.5$$

EXERCISE - 1.2

Textual

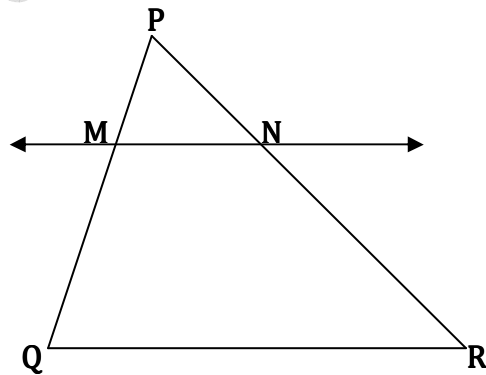
1. Line $MN \parallel$ side QR , If $PM = 4$, $MQ = 6$, $PN = 5$, then find NR .



EXERCISE - 1.2

Solutions

1. In fig., line $MN \parallel$ side QR , If $PM = 4$, $MQ = 6$, $PN = 5$, then find NR .

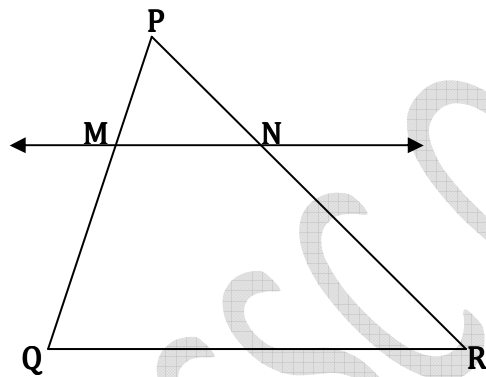


Solution :

Given : line $MN \parallel$ side QR ,

$$PM = 4, MQ = 6, PN = 5$$

Find : NR



In ΔPQR ,

$MN \parallel QR$ --- (Given)

$$\therefore \frac{PM}{MQ} = \frac{PN}{NR} \quad \text{--- (B.P.T)}$$

$$\therefore \frac{4}{6} = \frac{5}{NR}$$

$$\therefore NR = \frac{6 \times 5}{4} = \frac{15}{2} = 7.5$$

$$\therefore NR = 7.5$$

PROBLEM SET – 1

Textual

1. ΔABC and ΔPQR have same base and their corresponding heights are 5 and 3.5 units. Find $\frac{A(\Delta ABC)}{A(\Delta PQR)}$

PROBLEM SET – 1

Solutions

1. ΔABC and ΔPQR have same base and their corresponding heights are 5 and 3.5 units. Find $\frac{A(\Delta ABC)}{A(\Delta PQR)}$

Given : ΔABC and ΔPQR have same base and heights are 5 and 3.5 units.

Find : $\frac{A(\Delta ABC)}{A(\Delta PQR)}$

Solution :

$$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{b \times 5}{b \times 3.5} = \frac{10}{7}$$

$$\therefore \frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{10}{7}$$

If the bases are same the areas are in proportion to their corresponding heights.

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